MATHEMATICS

WORK SHEET-1
(Trigonometry Equation)

Subjective Questions

1. Solve the equation for x,

$$5^{\frac{1}{2}}+5^{\frac{1}{2}+\log_5(\sin x)}=15^{\frac{1}{2}+\log_{15}\cos x}$$

2. Find all the values of θ satisfying the equation; $\sin\theta+\sin 5\theta=\sin 3\theta$ such that $0\le\theta\le\pi$.

3. Solve the equality: $2\sin 11x+\cos 3x+\sqrt{3}\sin 3x=0$

4. Find all value of θ, between 0 & π, which satisfy the equation; $\cos\theta.\cos 2\theta.\cos 3\theta=1/4$.

5. Solve for x, the equation $\sqrt{13-18\tan x}=6\tan x-3$, where $-2\pi<x<2\pi$.

6. Determine the smallest positive value of x which satisfy the equation, $\sqrt{1+\sin 2x}-\sqrt{2}\cos 3x=0$

7. Find the number of principal solution of the equation, $\sin x-\sin 3x+\sin 5x=\cos x-\cos 3x+\cos 5x$.

8. Find the general solution of the trigonometric equation

$$3^{\left(\frac{1}{2}+\log_3(\cos x+\sin x)\right)}-2^{\log_2(\cos x-\sin x)}=\sqrt{2}$$

9. Find all values of θ between 0° & 180° satisfying the equation ; $\cos 6\theta+\cos 4\theta+\cos 2\theta+1=0$.

10. Find the general solution of the equation, $\sin\pi x+\cos\pi x=0$. Also find the sum of all solutions in [0, 100].

11. Find the range of y such that the equation, $y+\cos x=\sin x$ has a real solution. For $y=1$, find x such that $0<x<2\pi$

12. Find the general values of θ for which the quadratic function $(\sin\theta)x^2+(2\cos\theta)x+\dfrac{\cos\theta+\sin\theta}{2}$ is the square of a linear function.

13. Prove that the equations

(a) $\sin x\cdot\sin 2x\cdot\sin 3x=1$

(b) $\sin x\cdot\cos 4x\cdot\sin 5x=-1/2$ have no solution.

14. Let $f(x)=\sin^6x+\cos^6x+k(\sin^4x+\cos^4x)$ for some real number k. Determine

(a) all real numbers k for which $f(x)$ is constant for all values of x.

(b) all real numbers k for which there exists a real number 'c' such that $f(c)=0$.

(c) If $k=-0.7$, determine all solutions to the equation $f(x)=0$.

15. If the set of values of x satisfying the inequality $\tan x.\tan 3x<-1$ in the interval $\left(0,\frac{\pi}{2}\right)$ is (a, b), then the value of $\left(\frac{36(b-a)}{\pi}\right)$ is

16. Solve for x, $(-\pi\le x\le\pi)$ the equation; $2(\cos x+\cos 2x)+\sin 2x(1+2\cos x)=2\sin x$.

17. Find the general solution of the following equation : $2(\sin x-\cos 2x)-\sin 2x(1+2\sin x)+2\cos x=0$.

18. Find the values of x, between 0 & 2π, satisfying the equation $\cos 3x+\cos 2x=\sin\frac{3x}{2}+\sin\frac{x}{2}$

19. Solve: $\tan^2 2x+\cot^2 2x+2\tan 2x+2\cot 2x=6$.

20. Find the set of values of x satisfying the equality $\sin\left(x-\frac{\pi}{4}\right)-\cos\left(x+\frac{3\pi}{4}\right)=1$ and the inequality $\dfrac{2\cos 7x}{\cos 3+\sin 3}>2^{\cos 2x}$

WORK SHEET-2 (Limit and differentiation)

Subjective Questions

1. $$\lim_{x\to 1}\frac{\left[\sum_{K=1}^{100}x^k\right]-100}{x-1}$$

2. $$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{1-\sqrt{2}\sin x}$$

3. $$\lim_{x\to 0}\frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$$

4. $\lim\limits_{\theta\to\frac{\pi}{4}}\dfrac{\sqrt{2}-\cos\theta-\sin\theta}{(4\theta-\pi)^2}$

5. $\lim\limits_{h\to 0}\dfrac{\sin\left(\frac{\pi}{3}+4h\right)-4\sin\left(\frac{\pi}{3}+3h\right)+6\sin\left(\frac{\pi}{3}+2h\right)-4\sin\left(\frac{\pi}{3}+h\right)+\sin\frac{\pi}{3}}{h^4}$

6. $\lim\limits_{x\to\infty}x^2\left(\sqrt{\dfrac{x+2}{x}}-\sqrt[3]{\dfrac{x+3}{x}}\right)$

7. $\lim\limits_{x\to-\infty}\dfrac{\left(3x^4+2x^2\right)\sin\frac{1}{x}+|x|^3+5}{|x|^3+|x|^2+|x|+1}$

8. If $\ell=\lim\limits_{n\to\infty}\sum\limits_{r=2}^{n}\left((r+1)\sin\dfrac{\pi}{r+1}-r\sin\dfrac{\pi}{r}\right)$ then find $\{\ell\}$. (where { } denotes the fractional part function)

9. Find a and b if : (i) $\lim\limits_{x\to\infty}\left[\dfrac{x^2+1}{x+1}-ax-b\right]=0$

(ii) $\lim\limits_{x\to-\infty}\left[\sqrt{x^2-x+1}-ax-b\right]=0$

10. $\lim\limits_{x\to 0}$ **[**$\ell n\,(1+\sin^2x).\cot\,(\ell n^2\,(1+x))$**]**

11. (a) $\lim\limits_{x\to 0}\ \tan^{-1}\dfrac{a}{x^2}$, where $a\in R$; (b) Plot the graph of the function $f(x)=\lim\limits_{t\to 0}\left(\dfrac{2x}{\pi}\tan^{-1}\dfrac{x}{t^2}\right)$

12. Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$be sequences such that
(i) $a_n+b_n+c_n=2n+1$; (ii) $a_nb_n+b_nc_n+c_na_n=2n-1$;
(iii) $a_nb_nc_n=-1$; (iv) $a_n<b_n<c_n$
Then find the value of $\lim\limits_{n\to\infty}(na_n)$.

13. Let $f(x)=ax^3+bx^2+cx+d$ and $g(x)=x^2+x-2$. If $\lim\limits_{x\to 1}\dfrac{f(x)}{g(x)}=1$ and $\lim\limits_{x\to -2}\dfrac{f(x)}{g(x)}=4$, then find the value of $\dfrac{c^2+d^2}{a^2+b^2}$.

14. $\lim\limits_{x\to\infty}\left(\dfrac{x+c}{x-c}\right)^x=4$ then find c

15. If $n\in N$ and $a_n=2^2+4^2+6^2+.......+(2n)^2$ and $b_n=1^2+3^2+5^2+......+(2n-1)^2$.

Find the value $\lim\limits_{n\to\infty}\dfrac{\sqrt{a_n}-\sqrt{b_n}}{\sqrt{n}}$.

16. $\lim\limits_{n\to\infty}\left(\dfrac{\sqrt{n^2+n}-1}{n}\right)^{2\sqrt{n^2+n}-1}$

17. $\lim\limits_{x\to\infty}\left(\dfrac{a_1^{\frac{1}{x}}+a_2^{\frac{1}{x}}+a_3^{\frac{1}{x}}+.......+a_n^{\frac{1}{x}}}{n}\right)^{nx}$, where $a_1, a_2, ...a_n>0$

18. $\lim\limits_{x\to 0}\left[\dfrac{(1+x)^{1/x}}{e}\right]^{1/x}$

19. If $\lim\limits_{x\to\infty}\dfrac{a(2x^3-x^2)+b(x^3+5x^2-1)-c(3x^3+x^2)}{a(5x^4-x)-bx^4+c(4x^4+1)+2x^2+5x}=1$, then the value of $(a+b+c)$ can be expressed in the lowest form as $\dfrac{p}{q}$. Find the value of $(p+q)$.

20. $\lim\limits_{x\to 0}\left[\dfrac{\ell n(1+x)^{1+x}}{x^2}-\dfrac{1}{x}\right]$

21. Let $L=\prod\limits_{n=3}^{\infty}\left(1-\dfrac{4}{n^2}\right)$; $M=\prod\limits_{n=2}^{\infty}\left(\dfrac{n^3-1}{n^3+1}\right)$ and $N=\prod\limits_{n=1}^{\infty}\dfrac{(1+n^{-1})^2}{1+2n^{-1}}$, then find the value of $L^{-1}+M^{-1}+N^{-1}$.

22. Let $f(x)=\lim\limits_{n\to\infty}\sum\limits_{n=1}^{n}3^{n-1}\sin^3\dfrac{x}{3^n}$ and $g(x)=x-4f(x)$. Evaluate $\lim\limits_{x\to 0}(1+g(x))^{\cot x}$.

23. If $f(n,\theta)=\prod\limits_{r=1}^{n}\left(1-\tan^2\dfrac{\theta}{2^r}\right)$, then compute $\lim\limits_{n\to\infty}f(n,\theta)$

24. Evaluate $\lim\limits_{x\to\infty}\left(\dfrac{x}{e}-x\left(\dfrac{x}{x+1}\right)^x\right)$

25. $f(x)$ is the function such that $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=1$.

$\lim\limits_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{(f(x))^3}=1$, then find the value of a and b.

26. Through a point A on a circle, a chord AP is drawn and on the tangent at A a point T is taken such that AT = AP. If TP produced meet the diameter through A at Q, prove that the limiting value of AQ when P moves upto A is double the diameter of the circle.

27. At the end points A, B of the fixed segment of length L, lines are drawn meeting in C and making angles θ and 2θ respectively with the given segment. Let D be the foot of the altitude CD and let x represents the length of AD. Find the value of x as θ tends to zero i.e. $\lim\limits_{\theta\to 0} x$.

WORK SHEET-3 (Complex Number)

Subjective Questions

1. Number of values of z (real or complex) simultaneously satisfying the system of equations $1 + z + z^2 + z^3 + ... + z^{17} = 0$ and $1 + z + z^2 + z^3 + ... + z^{13} = 0$ is

2. Let $Z_1 = (8 + i)\sin\theta + (7 + 4i)\cos\theta$ and $Z_2 = (1 + 8i)\sin\theta + (4 + 7i)\cos\theta$ are two complex numbers. If $Z_1 \cdot Z_2 = a + ib$ where $a, b \in R$ then the largest value of $(a + b)$ $\forall\, \theta \in R$, is

3. If z_1 & $\bar{z}_1$ represent adjacent vertices of a regular polygon of n sides with centre at the origin & if $\frac{\operatorname{Im} z_1}{\operatorname{Re} z_1} = \sqrt{2} - 1$ then the value of n is equal to :

4. If z is a complex number satisfying the equation $|z + i| + |z - i| = 8$, on the complex plane then maximum value of $|z|$ is

5. Let Z be a complex number satisfying the equation $(Z^3 + 3)^2 = -16$ then $|Z|$ has the value equal to

6. The area of the triangle whose vertices are the roots $z^3 + iz^2 + 2i = 0$ is

7. Given $z_p = \cos\left(\frac{\pi}{2^P}\right) + i\sin\left(\frac{\pi}{2^P}\right)$, then $\lim\limits_{n\to\infty}(z_1 z_2 z_3 ....z_n) =$

8. If z_1, z_2, z_3 are 3 distinct complex numbers such that $\frac{3}{|z_2 - z_3|} = \frac{4}{|z_3 - z_1|} = \frac{5}{|z_1 - z_2|}$, then the value of $\frac{9}{z_2 - z_3} + \frac{16}{z_3 - z_1} + \frac{25}{z_1 - z_2}$ equals

9. Let Z is complex satisfying the equation $z^2 - (3 + i)z + m + 2i = 0$, where $m \in R$. Suppose the equation has a real root. The additive inverse of non real root, is

10. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

11. $z_1 = \frac{a}{1-i}$; $z_2 = \frac{b}{2+i}$; $z_3 = a - bi$ for $a, b \in R$ if $z_1 - z_2 = 1$ then the centroid of the triangle formed by the points z_1, z_2, z_3 in the argand's plane is given by

12. Number of complex numbers z such that $|z| = 1$ and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ is

13. If P and Q are respectively by the complex numbers z_1 and z_2 such that $\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = \left|\frac{1}{z_1} - \frac{1}{z_2}\right|$, then the circumcentre of ΔOPQ (where O is the origin) is

14. If z_1 & z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\operatorname{Arg} z_1 - \operatorname{Arg} z_2$ is equal to

15. If the complex number z satisfies the condition $|z| \geq 3$, then the least value of $\left|z + \frac{1}{z}\right|$ is equal to:

16. If $\cos\theta + i\sin\theta$ is a root of the equation $x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$ then the value of $\sum_{r=1}^{n} a_r \cos r\theta$ equals (where all coefficient are real)

WORK SHEET- 4 (Quadratic Equation)

Subjective Questions

1. Find the value of a for which one root of the equation $x^2 + (2a - 1)x + a^2 + 2 = 0$ is twice as large as the other.

2. Find a such that one of the roots of the equation $x^2 - \frac{15}{4}x + a = 0$ is the square of the other.

3. Find k in the equation $5x^2 - kx + 1 = 0$ such that the difference between the roots of the equation is unity.

4. Find b in the equation $5x^2 + bx - 28 = 0$ if the roots x_1 and x_2 of the equation are related as $5x_1 + 2x_2 = 1$ and b is an integer.

5. Find the values of the coefficient a for which the curve $y = x^2 + ax + 25$ touches the OX axis.

6. For what values of p does the vertex of the parabola $y = x^2 + 2px + 13$ lie at a distance of 5 from the origin ?

7. If x_1, x_2 are the roots of $ax^2 + bx + c = 0$, then find the value of
(i) $(ax_1 + b)^{-2} + (ax_2 + b)^{-2}$,
(ii) $(ax_1 + b)^{-3} + (ax_2 + b)^{-3}$.

8. If α, β are the roots of $ax^2 + bx + c = 0$, $(a \neq 0)$ and $\alpha + \delta$, $\beta + \delta$ are the roots of $Ax^2 + Bx + C = 0$, $(A \neq 0)$ for some constant δ, then prove that, $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4ac}{A^2}$.

9. Solve following Inequalities over the set of real numbers -

(i) $\frac{x^2 + 2x - 3}{x^2 + 1} < 0$ (ii) $\frac{(x-1)(x+2)^2}{-1-x} < 0$

(iii) $x^4 - 2x^2 - 63 \leq 0$ (iv) $\frac{x+1}{(x-1)^2} < 1$

(v) $\frac{x^2 - 7x + 12}{2x^2 + 4x + 5} > 0$ (vi) $\frac{x^2 + 6x - 7}{x^2 + 1} \leq 2$

(vii) $\frac{x^4 + x^2 + 1}{x^2 - 4x - 5} < 0$ (viii) $\frac{x+7}{x-5} + \frac{3x+1}{2} \geq 0$

(ix) $\frac{1}{x+2} < \frac{3}{x-3}$ (x) $\frac{14x}{x+1} - \frac{9x-30}{x-4} < 0$

(xi) $\frac{x^2 - 5x + 12}{x^2 - 4x + 5} > 3$ (xii) $\frac{x^2 + 2}{x^2 - 1} < -2$

(xiii) $\frac{(2 - x^2)(x-3)^3}{(x+1)(x^2 - 3x - 4)} \geq 0$

(xiv) $\frac{5 - 4x}{3x^2 - x - 4} < 4$

(xv) $\frac{(x+2)(x^2 - 2x + 1)}{4 + 3x - x^2} \geq 0$

(xvi) $\frac{x^4 - 3x^3 + 2x^2}{x^2 - x - 30} > \frac{1}{x}$

(xvii) $\frac{2x}{x^2 - 9} \leq \frac{1}{x+2}$

(xviii) $\frac{1}{x-2} + \frac{1}{x-1} > \frac{1}{x}$

(xix) $\frac{20}{(x-3)(x-4)} + \frac{10}{x-4} + 1 > 0$

(xx) $\frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1$

(xxi) $(x^2 - 2x)(2x - 2) - 9\frac{2x-2}{x^2 - 2x} \leq 0$

10. Let the quadratic equation $x^2 + 3x - k = 0$ has roots a, b and $x^2 + 3x - 10 = 0$ has roots c, d such that modulus of difference of the roots of the first equation is equal to twice the modulus of the difference of the roots of the second equation. If the value of 'k' can be expressed as rational number in the lowest form as m n then find the value of (m + n).

11. We call 'p' a good number if the inequality $\frac{2x^2 + 2x + 3}{x^2 + x + 1} \leq p$ is satisfied for any real x. Find the smallest integral good number.

12. For what values of m will the expression $y^2 + 2xy + 2x + my - 3$ be capable of resolution into two rational factors ?

13. Find the complete set of real values of 'a' for which both roots of the quadratic equation $(a^2 - 6a + 5)x^2 - \sqrt{a^2 + 2a}\,x + (6a - a^2 - 8) = 0$ lie on either side of the origin.